This document contains edited copy of short papers describing the basis of the matrix approach for 3D kinematic and dynamic analysis which is at the basis of the software library SPACELIB.

More details can be found in:

Legnani G, Casolo F., Righettini P., Zappa B. <A Homogeneous Matrix Approach to 3D Kinematics and Dynamics. ***Part 1: theory***>. Mechanisms and Machine Theory (the scientific journal of IFToMM. Pergamon Press U.K.) vol.31, n.5, pp.573-587, 1996

Legnani G, Casolo F., Zappa B, Righettini P. <A Homogeneous Matrix Approach to 3D Kinematics and Dynamics. ***Part 2: applications***. Mechanisms and Machine Theory (the scientific journal of IFToMM. Pergamon Press U.K.) vol.31, n.5, pp.589-605, 1996

G. Legnani, “Robotica Industriale" Casa Editrice Ambrosiana, giugno 2003, isbn-88-408-1262-8

a wide bibliography list can be found in the SPACELIB reference manual.

Notation

vector   matrix notation 1   matrix notation2

$$\vec{v} \qquad v = \begin{vmatrix} v_x \\ v_y \\ v_z \end{vmatrix} \qquad \underline{v} = \begin{vmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{vmatrix}$$

cross product

$$\vec{a} = \vec{b} \times \vec{c} \qquad a = \underline{b}c = -\underline{c}b \qquad \underline{a} = \underline{b}\underline{c} - \underline{c}\underline{b} = -bc^t + cb^t$$

XII International Congress of Biomechanics ISB
26-30 June 1989
University of California, Los Angeles, USA

A NEW APPROACH TO IDENTIFY KINEMATIC PECULIARITIES IN HUMAN MOTION

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ABSTRACT

A new matrix method recently developed by the authors, suitable for an easy recognition of the kinematic characteristics of the human motion, is here presented. The method is based on a coherent use of 4x4 matrices, and beside the usual homogeneous transformation matrix (here called position matrix) it implies the use of three new matrices that are: the velocity W, the acceleration H and the generalized speed ratio L matrices. A peculiar characteristic of these matrices is that they contain both the angular and linear terms (e.g. W includes both the linear and angular velocities of a body). Some features of this methodology are presented by means of some practical examples concerning human motion analysis. They are as follows: the recognition of instantaneous screw axis, the location of the center of accelerations and the identification of the best equivalent

joint of a human articulation. In this case the norm of a certain matrix ΔL provides a measure of the error that is introduced with the choice of a certain joint-model.

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1 - INTRODUCTION.

The procedure presented below, which is based on a new methodology recently developed by the authors [1] to study kinematic chains of rigid bodies, allows an easy identification of some kinematic peculiarities of the human body motion.

2 - METHOD.

Our method, beside the well-known homogeneous transformation matrices that we call position matrices M, introduces other 4x4 matrices:

a) the speed matrix W containing both the speed and the angular speed of a body;
b) the acceleration matrix H, containing both the acceleration and the angular acceleration;
c) the generalized speed ratio matrix L, describing the Instantaneous Screw Axis (ISA) between two bodies.

These matrices allow us to handle linear and angular quantities simultaneously, simplifying the usual kinematic relations. Moreover, since these matrices are cartesian representations of tensors, this approach allows the preliminary development and the symbolic handling of the kinematic relations out of any reference frame. The mentioned matrices are

M_{ij} position matrix of frame j with respect to frame i
R rotation matrix

$$M_{ij} = \begin{vmatrix} Xx & Yx & Zx & t_X \\ Xy & Yy & Zy & t_y \\ Xz & Yz & Zz & t_Z \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} & R & & T \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$M_{ji} = M_{ij}^{-1} = \begin{vmatrix} R^t & -R^tT \\ 0\quad 0\quad 0 & 0 \end{vmatrix} \qquad R^{-1} = R^t \qquad \text{matrix inversion}$$

$$w_{ij(k)} = \begin{vmatrix} 0 & -w_Z & w_Y & v_X \\ w_Z & 0 & -w_X & v_y \\ -w_Y & w_X & 0 & v_Z \\ 0 & 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} & \underline{w} & & v_0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$L_{ij(k)} = \begin{vmatrix} 0 & -u_Z & u_Y & b_X \\ u_Z & 0 & -u_X & b_y \\ -u_Y & u_X & 0 & b_Z \\ 0 & 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} & \underline{u} & & B \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$U = [u_x \quad u_y \quad u_z]^T \qquad T = [t_x \quad t_y \quad t_z]^T \qquad B = [b_x \quad b_y \quad b_z]^T \qquad P = [p_x \quad p_y \quad p_z]^T$$

$$L_{ij(k)} = \begin{cases} W_{ij(k)} / |w| & if \quad w \neq 0 \\ W_{ij(k)} / |V_0| & if \quad w = 0 \end{cases} \qquad B = -\underline{U}P + pU$$

ω is the relative angular velocity between the bodies i and j, V_0 is the relative velocity of two particular points [1] of the two bodies, U and P are the unit vector and a point of the ISA respectively; finally p is the pitch of the screw. All the quantities are expressed with respect to the frame (k). The matrix H, which represents the relative acceleration of two bodies, is obtained by adding the square of W to its time derivative W'. Speed and acceleration matrices are related to the position matrix, as well as to its first and second time derivatives M', M'' by the relations [1,2]

$$(1) \qquad \begin{cases} W_{ij(i)} = M'_{ij(i)} M^{-1}_{ij(i)} \\ H_{ij(i)} = M''_{ij(i)} M^{-1}_{ij(i)} \end{cases} \qquad H_{ij(k)} = W'_{ij(k)} + W^2_{ij(k)} = \left|\begin{array}{ccc:c} & \underline{G} & & A_0 \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right|$$

The following formulas show the relations linking the representations of the above mentioned quantities W, L and H into different frames (r and s)

$$W_{ij(r)} = M_{rs} W_{ij(s)} M^{-1}_{rs} \quad L_{ij(r)} = M_{rs} L_{ij(s)} M^{-1}_{rs} \quad H_{ij(r)} = M_{rs} H_{ij(s)} M^{-1}_{rs}$$

3-APPLICATIONS.

The following examples show how our method can be easily applied to practical biomechanical problems using (for instance) space_lib, a software library, described in [5].

3.1 - VELOCITY AND ACCELERATION ANALYSIS.

Let us consider the kinematic analysis of an athlete executing a gymnastic exercise. Several experimental techniques are available to evaluate the successive positions (and hence the M matrices) of the body segments. The next step is generally the speed and the acceleration calculation. This operation, that consists in the evaluation of the matrices W and H, can be efficiently executed remembering the eq.s (1) and numerically calculating M' and M''

$$(2) \qquad M'_{<t>} \cong \frac{M_{<t+dt>} - M_{<t-dt>}}{2dt} \qquad M''_{<t>} \cong \frac{M_{<t+dt>} - M_{<t>} + M_{<t-dt>}}{dt^2}$$

The evaluation of $W_{<t>}$ and $H_{<t>}$ through the use of the simple formulas (1) does not present numerical problems because the inversion of the matrices M can be done without the execution of any division.

3.2 - IDENTIFICATION OF ISA.

The ISA can be easily evaluated by the matrix L that is directly obtained from W. In fact the unit vector U is already contained in L while the pitch p and the point P can be easily evaluated as:

$p=U^t B$; $P=\underline{U}B$. (in fact $U^t \underline{U}=0$, $\underline{U}U^t=0$, $U^t U=1$ and the pseudo-inverse of U is -U)

3.3 - TESTING THE JOINT MODELS.

As an example, we now analyze the problem to evaluate the error which is introduced when a given human articulation is considered as a revolute pair. This task can be performed analyzing (during the execution of a movement) the locus of the ISA between the two parts of the body which are connected by the articulation. If the considered parts of the body were connected by a revolute pair, the matrix L would be constant and the pitch would be null. Since the human articulations are not exactly revolute pairs, L will never be constant and the pitch of the ISA will never be null.

An index of the variability of L is the norm ε of DL:

$$e = \max(\|\Delta L\|) = \max(\|L_{<t>} - L^*\|)$$

where $L_{<t>}$ is the instantaneous value of L and L^* is its average value.

Moreover, the average value p^* of the pitch can be extracted from L^*.

Therefore the values of ε and p^* are proportional to the error that would be introduced with the substitution of the articulation by a revolute pair.
Practically, a joint can be approximated with a revolute pair when both ε and p^* are smaller than two pre-established values.
Approximated spherical pairs can be tested with similar procedures.

3.4 - IDENTIFICATION OF ACCELERATION CENTER.

The acceleration center P^* of a rigid body can be easily evaluated, if it exists, by the following formula [3]
$P^* = G^{-1} A_0$ where G and A_0 belong to the acceleration matrix H.

4 - DYNAMICS.

The extension of this method to the dynamics analysis of human motion is presented in [4].

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[1] C.Casolo, G.Legnani "A Consistent Matrix Approach for the Kinematics and Dynamics of Chains of Rigid Bodies" AIMETA Congress - Bari, Italy 1988.
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ABSTRACT

This work introduces a new matrix approach to the dynamics of human motion.
This approach is based on a method for the analysis of rigid body chains that has been recently developed by the authors and that makes a coherent use of 4x4 matrices. Beside M, W, H, that are position, velocity, and acceleration, matrices respectively, and that include both the linear and rotational components, three new matrices containing the dynamic parameters have been introduced.
Those are:
1) the action matrices Φ describing systems of forces and torques applied to the bodies;
2) the momentum matrices Γ that contains moment and angular moment of the bodies;
3) the pseudo-inertial matrices J describing the mass distribution.
This method is suitable for the automatic analysis of rigid bodies chains. The paper shortly describes how to apply this methodology to the human motion analysis. Floor gymnastics has been considered as an example. In particular, the matter discussed here concern how the trajectory of the whole body is affected by the relative motion of the athlete's limbs.

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METHODOLOGY.

This approach is based on a method for the analysis of rigid body chains recently developed by the authors [1,2,3] and makes a coherent use of 4×4 matrices. The resulting notation is succinct, expressive and convenient for the automatic calculation. Beside M, W, H, that are position, velocity and acceleration matrices respectively, and contain both the linear and the rotational components, three new matrices which include the dynamic parameters have been introduced as follows:

$$\Phi_{k(s)} = \left|\begin{array}{ccc:c} 0 & -c_z & c_y & f_x \\ c_z & 0 & -c_x & f_y \\ -c_y & c_x & 0 & f_z \\ \hdashline -f_x & -f_y & -f_z & 0 \end{array}\right| \quad \Gamma_{k(s)} = \left|\begin{array}{ccc:c} 0 & -g_z & g_y & r_x \\ g_z & 0 & -g_x & r_y \\ -g_y & g_x & 0 & r_z \\ \hdashline -r_x & -r_y & -r_z & 0 \end{array}\right| \quad J_{k(s)} = \left|\begin{array}{ccc:c} I_{xx} & I_{xy} & I_{xz} & q_x \\ I_{xy} & I_{yy} & I_{yz} & q_y \\ I_{xz} & I_{yz} & I_{zz} & q_z \\ \hdashline q_x & q_y & q_z & m \end{array}\right|$$

$$\Phi_{k(s)} = \left|\begin{array}{c:c} \underline{c} & f \\ \hdashline -f^t & 0 \end{array}\right| \quad \Gamma_{k(s)} = \left|\begin{array}{c:c} \underline{g} & r \\ \hdashline -r^t & 0 \end{array}\right| \quad J_{k(s)} = \left|\begin{array}{c:c} \underline{I} & Q \\ \hdashline Q^t & m \end{array}\right|$$

where: $Q = m\,[x_g \;\; y_g \;\; z_g]$

$$I_{xx} = \int x^2 dm \quad I_{yy} = \int y^2 dm \quad I_{zz} = \int z^2 dm$$

$$I_{xy} = \int xy\,dm \quad I_{yz} = \int yz\,dm \quad I_{zx} = \int zx\;dm$$

$\Phi_{k(s)}$ is the action matrix and contains, relatively to a frame (s), the components of the total force F and torque C applied to a body k. $\Gamma_{k(s)}$ is the momentum matrix of the body k with respect to a frame (s); ρ represents the linear momentum of the rigid body and γ is its angular momentum expressed in (s). $J_{k(s)}$ represents, in the reference (s), the mass distribution of a body k; m is its mass and $Q/m = [x_g \;\; y_g \;\; z_g]$ are the coordinates of its center of mass. The matrices $\Phi_{k(s)}$, $\Gamma_{k(s)}$ and $J_{k(s)}$ can be seen as the components (in the reference (s)) of the contravariant tensors Φ, Γ and J; thus, if their representation in any frame (r) is required, we may use the following formulas [1]

$$\Phi_{(r)} = M_{rs}\Phi_{(s)}M_{rs}^t \quad \Gamma_{(r)} = M_{rs}\Gamma_{(s)}M_{rs}^t \quad J_{(r)} = M_{rs}J_{(s)}M_{rs}^t$$

The dynamic equilibrium and the momentum equations of a body k may be succinctly written introducing the skew operator (skew{X} := $X - X^t$):

$$\Gamma_{(0)} = \text{skew}\{W\,J\} = WJ - JW^t; \qquad \Phi_{(0)} = \text{skew}\{H\,J\} = HJ - JH^t \qquad 0 = \text{inertial frame}$$

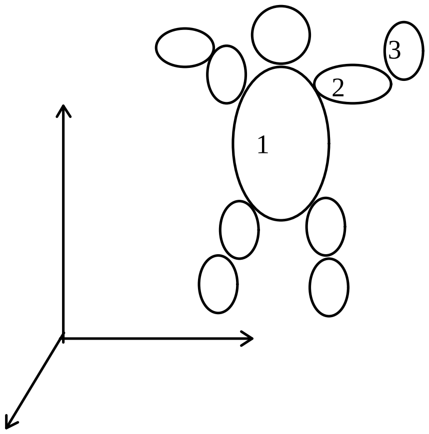
THE HUMAN MOVEMENT ANALYSIS.

To show a practical application of the method, let us analyze, as an example, the following problem: how, during a jump, a gymnast changes the orientation of his trunk moving his arms and legs (A software library suitable for the solution of this problem is shown in [4]).

To know the motion of limbs means to know the relative position matrices M_{ij}, as well as the velocity W_{ij} and acceleration H_{ij} matrices of the body segments i and j. Therefore, we can evaluate the trajectory and the successive orientations of the athlete's trunk knowing the link motions plus the initial conditions (absolute position and speed) of the trunk and the inertial matrices J of the segments of the chain.

The absolute position M_{0i}, speed W_{0i} and acceleration H_{0i} of each part i of the human body can be evaluated using formulas like these [1], [2], [3]:

$$\begin{cases} M_{02} = M_{01}M_{12} \\ M_{03} = M_{02}M_{23} = M_{01}M_{12}M_{23} \\ \dots \end{cases} \quad \begin{cases} W_{02} = W_{01} + W_{12} \\ W_{03} = W_{02} + W_{23} = W_{01} + W_{12} + W_{23} \\ \dots \end{cases}$$



$$\begin{cases} H_{02} = H_{01} + 2W_{01}W_{12} + H_{12} = W_{01}^{'} + \tilde{H}_{02} \\ H_{03} = H_{02} + 2W_{02}W_{23} + H_{23} = W_{01}^{'} + \tilde{H}_{03} \\ \dots \end{cases} \qquad \boxed{H_{01} = W_{01}^{'} + W_{01}^{2}}$$

where each term $\tilde{H}_{0i}$ is the absolute acceleration of the body i, which is calculated considering the unknown part $W_{01}^{'}$ of the trunk acceleration as null. The equation of the dynamic equilibrium of the whole human body can be written as follows:

$$(1) \quad skew\left\{\sum_i \left(H_g J_i\right)\right\} = skew\left\{\sum_i \left(W_{01}^{'} J_i\right) + \sum_i \left(\tilde{H}_{0i} J_i\right) + \right\} \qquad H_{g(0)} = \left|\begin{array}{ccc:c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right|$$

The left side of eq. (1) is the total weight force and H_g is the gravity acceleration ($g=-9.81\ m/s^2$). Rearranging the terms of the differential eq.(1), it becomes:

$$(2) \ \boxed{\Phi^* = skew\left\{W_{01}^{'} J^*\right\}} \qquad \Phi^* = skew\left\{H_g J^* - \sum_i \left(\tilde{H}_{0i} J_i\right) + \right\} \qquad J^* = \sum_i J_i$$

where $W_{01}^{'}$ is the only unknown; the six elements of $W_{01}^{'}$ can be evaluated solving the following six-order linear system, that has been obtained by handling the matrix system (2):

$$\begin{cases}
\left(I_{yy} + I_{zz}\right)\dot{w}_x & - I_{xy}\dot{w}_y & - I_{xz}\dot{w}_z & & - mz_g a_y & + my_g a_z & = & c_x^* \\
- I_{xy}\dot{w}_x & + \left(I_{xx} + I_{zz}\right)\dot{w}_y & - I_{yz}\dot{w}_z & + mz_g a_x & & - mx_g a_z & = & c_y^* \\
- I_{xz}\dot{w}_x & - I_{yz}\dot{w}_y & + \left(I_{xx} + I_{yy}\right)\dot{w}_z & - my_g a_x & + mx_g a_y & & = & c_z^* \\
\hdashline
& mz_g \dot{w}_y & - my_g \dot{w}_z & ma_x & & & = & f_x^* \\
- mz_g \dot{w}_x & & mx_g \dot{w}_z & & ma_y & & = & f_y^* \\
my_g \dot{w}_x & - mx_g \dot{w}_y & & & & ma_z & = & f_z^*
\end{cases}$$

$$W'_{o1} = \left|\begin{array}{ccc:c} 0 & -\dot{w}_z & \dot{w}_y & a_X \\ \dot{w}_z & 0 & -\dot{w}_x & a_y \\ -\dot{w}_y & \dot{w}_x & 0 & a_Z \\ \hdashline 0 & 0 & 0 & 0 \end{array}\right| = \left|\begin{array}{c:c} \underline{\dot{w}} & a_0 \\ \hdashline 0 \quad 0 \quad 0 & 0 \end{array}\right|$$

The trajectories of the trunk can then be evaluated, for example, by means of the simple but efficient numerical integration algorithm:

$$\begin{cases} M_{01<t+dt>} \cong M_{01<t>} + M'_{01<t>}dt + \frac{1}{2} M''_{01<t>}dt^2 \\ W_{01<t+dt>} \cong W_{01<t>} + W'_{01<t>}dt \end{cases} \qquad \begin{cases} M'_{01} = W_{01} M_{01} \\ M''_{01} = H_{01} M_{01} = \left(W'_{01} + W^2_{01}\right) M_{01} \end{cases}$$

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[4] F.Casolo, R.Faglia, G.Legnani "Three Dimensional Analysis of Systems of Rigid Bodies" 10th DECUS Italian cong. April 1989, Baveno NO - Italy.

With ‘standard’ definition of the inertia moment eq.(2) would be written as:

$$\begin{cases} c = J\dot{w} + w \times Jw \\ f = ma \end{cases} \qquad J = \left|\begin{array}{ccc} J_x & J_{xy} & J_{xz} \\ J_{xy} & J_y & J_{yz} \\ J_{xz} & J_{yz} & J_z \end{array}\right|$$

$$J_x = I_{yy} + I_{zz} = \int \left(y^2 + z^2\right) dm$$

$$J_y = I_{xx} + I_{zz} = \int \left(x^2 + z^2\right) dm$$

$$J_z = I_{xx} + I_{yy} = \int \left(x^2 + y^2\right) dm$$

$$J_{xy} = -I_{xy} = -\int xy\, dm \quad J_{yz} = -I_{yz} = -\int yz\, dm \quad J_{xz} = -I_{xz} = -\int xz\, dm$$